Model of Intuitionistic Fuzzy Matrix Games based on Personal Wealth Voucher Data

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Abstract: For tracking of personal wealth vouchers and the sake of the problem of comparing order of fuzzy numbers in game theory based on the rational hypothesis of economic man. Therefore, the concept of intuitionistic element-order is advanced, and proved that it is the whole order. In this paper, it is proved that a real dual-matrix is correspond to the arbitrary fuzzy dual matrix, and also has the same Nash equilibrium solution with the arbitrary fuzzy dual matrix, and the solving of original problem is simplified, by fuzzy structured element theorem. Finally, an example is taken to illustrate effectivity.

1 Introduction

Game is a kind of system, which studies and predicts the decision of players' behavior with mathematical tools in a given strategy. Generally, the strategy is thought of exact values, but in real game situations, players cannot to make a decision exactly due to a lack of information and different understanding [1-2]. Therefore, lots of scholars have been investigating the topic with the fuzzy mathematic theory for portraying the reality better. Zhou [3], Song and Gao[4], Sun and Wang [5], Dey and Roy [6] used the game theory to solve the practical problems, Deng and Jiang [7], Deng and Jiang [8], Chavoshlou and Khamseh [9] studied the game theory with an uncertain strategy and exact payoffs. However, players exist hesitation except for affirmation and negation in real game with the change of environment, the progress of science and technology, political orientation, psychological behavior and ideological change etc. In other words, a fuzzy set, which is able to express degree of membership, degree of non-membership and hesitation degree, is necessary. In 1986, Atanassov [10] proposed the concept of an intuitionistic fuzzy set (IF-set), and hesitation degree is equal to 1 minus the sum of degree of membership and degree of non-membership. In 1994, P. Chen and Huang [11] introduced the concept of intuitionistic fuzzy number (IFN). Xu and Yager [12] introduced a static IFN. Guo [13] studied the dynamic IFN based on the fuzzy structured element. However, the matrix game is investigated less using the IF-set. Li and Nan [14-15] focus on the matrix game with payoffs of IF-set using interval IFN and triangular IFN (TIFN), respectively. Wan [16] studied the matrix game with payoffs of trapezoidal IFN. However, there exists 2 important problems to solute real game problems with IF theory, which are the parameter ergodic problem and the IFN ranking order problem. The first causes complex calculation, i.e., the traversal of all [0,1] interval. The second reflects inconformity of players' objective hypothesis, and less prominent of economic significance. Wan [17] and Guo [13] introduced the concept of fuzzy weight order, a total order. However, there is an essential difference between the two kinds of order. The first relies on the relationship of the IF membership function, and is not able to solve the problem of ergodicity preferably. The second has the characteristic of simple calculating and solving the ergodic problem. However, it is inadequate for rational decision of the economic man (if the same mean, the minimum variance strategy should be chosen). In this paper, we focus on reporting the solution method and calculating procedure for the matrix games with payoffs of TIFN. The concept of IFN order, which may express rational decision factors based on the fuzzy structured element theory, is introduced. IFN matrix games are formulated, and transformed classical matrix games based on the IFN order. Then a method is proposed to obtain optimal strategies for Players through solving a pair of bi-objective classical matrix games.

The rest of this paper is organized as follows. Section 2 introduces basic theories about IFN and fuzzy structured element. In Section 3 the IFN order is defined and proved. In Section 4 the methodology, from IFN matrix games to classical matrix games with payoffs of IFN, is proposed, and the procedure to solve the IF matrix games is define. In Section 5 and 6, a numerical example and concluding remarks are given, respectively.

2 IFN and Fuzzy Structured Element

2.1 IF-set and IFN

In this section, IF-set and IFN are defined as follows.

Definition 1[13]. A fuzzy set $\tilde{A}=\{<x,\mu_{\tilde{A}}(x),\nu_{\tilde{A}}(x)>|\, x\in X\}$ is a IF-set on a non empty set X, where the functions $\mu_{\tilde{A}}:X\to[0,1]$ and $\nu_{\tilde{A}}:X\to[0,1]$ represent the degree of membership and degree of non-membership, and $0\le\mu_{\tilde{A}}(x)+\nu_{\tilde{A}}(x)\le 1$. Let $\pi_{\underset{\sim}{A}}(x)=1-\mu_{\underset{\sim}{A}}(x)-\nu_{\underset{\sim}{A}}(x)$, which is called as the degree of hesitation as $x(x\in X)$ belongs to $\tilde{A}$.

Definition 2[13]. $\tilde{A}=<\tilde{\alpha},\tilde{\beta}>$, which has the characteristic function $<\mu_{\tilde{A}}(x),\nu_{\tilde{A}}(x)>$, is a IF-set on the real domain R. If $\tilde{\alpha}$ and $\tilde{\beta}^C$ are fuzzy numbers, $\tilde{A}$ is called the fuzzy number. All members of IF numbers are abbreviated to $I\tilde{F}S$.

Theorem 1[13]. $\tilde{A},\tilde{B}\in I\tilde{F}S$, and continuous operator is $\otimes$, then $\tilde{A}\otimes\tilde{B}\in I\tilde{F}S$.

2.2 Fuzzy structured element

Definition 3[18-19]. If fuzzy structured element E has the following properties, E is called to the regular fuzzy structured element.

(i) $\forall x\in(-1,1), E(x)>0$

(ii) $E(x)$ is continuous and increasing strictly on $[-1,0)$, $E(x)$ is continuous and decreasing strictly on $(0,1]$.

If $E(x)=E(-x)$, E is called to the symmetric fuzzy structured element.

Theorem 2[18-19]. Let E is a fuzzy structured element, if f and g are monotone and bounded functions, and then it is easily seen as $(f+g)(E)=f(E)+g(E)$.

2.3 The fuzzy structured element expression of IFN and weighted order

Theorem 3[13]. Let E is any fuzzy structured element, any IFN $\tilde{A}=<\tilde{\alpha},\tilde{\beta}>$ can be denoted by $\tilde{A}=<f(E),[g(E)]^C>$ with $\tilde{\alpha}=f(E),\tilde{\beta}^C=g(E)$, where $f(x)$ and $g(x)$ are same sequence of monotone and bounded functions on $[-1,1]$.

Definition 4[13]. Let $\tilde{A},\tilde{B}\in I\tilde{F}N$, denoted by $\hat{A}=<f_1(E),[g_1(E)]^C>$ and $\tilde{B}=<f_2(E),[g_2(E)]^C>$ with E which is a regular fuzzy structured element and has the membership function of $E(x)$. If f_1,f_2,g_1 and g_2 are the same sequence of monotone functions on $[-1,1]$, typically, the IFN weighted order is defined as follows: $\delta(\tilde{A},\tilde{B})=\int_{-1}^{1}E(x)((f_1(x)-f_2(x))+(g_1(x)-g_2(x)))dx$.

3 IFN Structured Order

Definition 5. Let $\tilde{A},\tilde{B}\in I\tilde{F}N$, denoted by $\tilde{A}=<f_1(E),[g_1(E)]^C>$ and $\tilde{B}=<f_2(E),[g_2(E)]^C>$, where with E which is a regular fuzzy structured element and has the membership function of $E(x)$. If f_1, f_2, g_1 and g_2 are the same sequence of monotone functions on $[-1,1]$, typically. The IFN order is defined as follows: $\delta(\tilde{A},\tilde{B})=\frac{1}{2}\int_{-1}^{1}(f_1(x)-f_2(x))+(g_1(x)-g_2(x)))(1-x)dx$, which is a total order with defining $\succ,\prec,\simeq$, and called IFN structured order.

Property 1. IFN structured order is a total order.

Prove.

(i) Reflexivity: $\delta(\tilde{A},\tilde{A})=\frac{1}{2}\int_{-1}^{1}(f_1(x)-f_1(x))+(g_1(x)-g_1(x))(1-x)dx \Leftrightarrow \tilde{A}=\tilde{A}$.

(ii) Antisymmetric:

$\tilde{A}\succ\tilde{B}$ and $\tilde{B}\prec\tilde{A}$ $\Leftrightarrow \delta(\tilde{B},\tilde{A})=\frac{1}{2}\int_{-1}^{1}((f_2(x)-f_1(x))+(g_2(x)-g_1(x)))(1-x)dx<0$,

and $\delta(\tilde{A},\tilde{B})=\frac{1}{2}\int_{-1}^{1}((f_1(x)-f_2(x))+(g_1(x)-g_2(x)))(1-x)dx=-\delta(\tilde{A},\tilde{B})<0$, $\delta(\tilde{A},\tilde{B})=0\Leftrightarrow\tilde{A}\simeq\tilde{B}$.

(iii) Transitivity:

$\tilde{C}\prec\tilde{B}$ and $\tilde{B}\prec\tilde{A}\Leftrightarrow\delta(\tilde{C},\tilde{B})\prec 0$ and $\delta(\tilde{B},\tilde{A})\prec 0$. $\tilde{C}\simeq\tilde{B}$ and $\tilde{B}\simeq\tilde{A}\Leftrightarrow\delta(\tilde{C},\tilde{B})\simeq 0$ and $\delta(\tilde{B},\tilde{A})\simeq 0$.

$$\begin{aligned}\delta(\tilde{C},\tilde{A})&=\frac{1}{2}\int_{-1}^{1}(((f_3(x)-f_1(x))+(g_3(x)-g_1(x)))(1-x)dx\\&=\frac{1}{2}\int_{-1}^{1}(f_3(x)-f_2(x)+f_2(x)-f_1(x))(1-x)dx+\frac{1}{2}\int_{-1}^{1}(g_3(x)-g_2(x)+g_2(x)-g_1(x))(1-\nu_{\tilde{A}}x)dx\\&=\delta(\tilde{C},\tilde{B})+\delta(\tilde{B},\tilde{A})<0\end{aligned}$$

In other words, $\delta(\tilde{C},\tilde{A})\prec 0$. And because of $\delta(\tilde{B},\tilde{A})=-\delta(\tilde{A},\tilde{B})$, it can be clear that $\delta(\tilde{B},\tilde{A})\prec 0$ or $\delta(\hat{A},\tilde{B})\prec 0$ as for $\tilde{A},\tilde{B}$.

Empathy may permit, $\delta(\tilde{C},\tilde{A})\simeq 0$.

Above all, $\succ,\prec,\simeq$ are the total orders on $I\tilde{F}N$.

$\tilde{A}_i=<\tilde{\alpha}_i,\tilde{\beta}_i>$ is called TIFN, which has the characteristic function of $<\mu_{\tilde{A}_i}(x),\nu_{\tilde{A}_i}(x)>$, where

$$\mu_{\tilde{A}_i}(x)=\begin{cases}1+\dfrac{x-b_i}{a_i}, b_i-a_i\le x\le b_i;\\ 1-\dfrac{x-b_i}{c_i}, b_i<x\le b_i+c_i;\\ 0, \text{otherwise.}\end{cases}$$, $\nu_{(\tilde{A}_i)^C}(x)=\mu_{\tilde{A}_i}(x)+\pi_{\tilde{A}_i}(x)$, with

$0\le\mu_{\tilde{A}_i}\le 1, 0\le\nu_{\tilde{A}_i}\le 1, \mu_{\tilde{A}_i}+\nu_{\tilde{A}_i}\le 1; a_i,b_i,c_i,d_i\in R$.

The order defined by Definition 4 and Literature 10 is a total order and has better capability of discrimination. However, it cannot distinguish the order of IFN with the membership function of $x\simeq K^*$. As shown in Fig.1. Assume that $\tilde{A},\tilde{B}$ be two TIFN with the meaning of profits and uniform probability distributions, the means of $\tilde{A},\tilde{B}$ are all K^*. The fluctuation of $\tilde{B}$ is more obvious than $\tilde{A}$, in other words, the variance of $\tilde{B}$ is bigger than $\tilde{A}$. Then as for the rational principle of economic man, the conclusion is easily reached as follows: $\tilde{B}\prec\tilde{A}$ or $\tilde{B}\simeq\tilde{A}$. However, the conclusions are different from Definition 4 and Literature 10, typically. And the following conclusion, which meets he rational principle of economic man, is valid $\tilde{B}\prec\tilde{A}$ or $\tilde{B}\simeq\tilde{A}$, based on Definition 5.

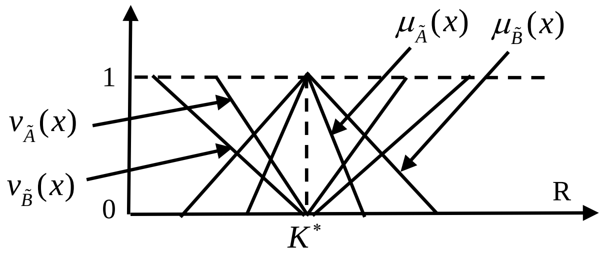

Fig. 1 The membership function of IFN

4 The Resolution of IF Matrix Games

The resolution of classical double matrix games is defined as follows:

Assume four tuple $G=(S^m,S^n,A_{m\times n},B_{m\times n})$ be a double matrix game, and a pair of array $(x^*,y^*)\in(S^m,S^n)$ be a rational strategy of double matrix games, with characteristics defined as follows:

(i) if $x^{\mathrm{T}}A_{m\times n}y^*\le x^{*\mathrm{T}}A_{m\times n}y^*,\forall x\in S^m,x\ge 0^n,x^{*\mathrm{T}}B_{m\times n}y\le x^{*\mathrm{T}}B_{m\times n}y^*,\forall y\in S^n,y\ge 0^n$, the values of Player I and Player II are $x^{*\mathrm{T}}A_{m\times n}y^*$ and $x^{*\mathrm{T}}B_{m\times n}y^*$ on $G=(S^m,S^n,A_{m\times n},B_{m\times n})$, typically. The optimum strategy is $(x^*,y^*)\in(S^m,S^n)$ on $G=(S^m,S^n,A_{m\times n},B_{m\times n})$.

Definition 6. Let $\tilde{A}_{m\times n}=(\tilde{a}_{ij})_{m\times n}$ be a IFN matrix game, where $\tilde{a}_{ij}=<\tilde{\alpha}_{ij},\tilde{\beta}_{ij}>$, $\tilde{\alpha}_{ij},\tilde{\beta}_{ij}\in IF\tilde{N}$, and $\tilde{\alpha}_{ij}=f_{ij}(E),\tilde{\beta}^C_{ij}=g_{ij}(E)$. If $f_{ij}(x)$ and $g_{ij}(x)$ are two functions same sequencing, bounding and monotoning on $[-1,1]$, typically, then it is called $\tilde{A}$ is a IFN monotone matrix; if $f_{ij}(x)$ and $g_{ij}(x)$ are the functions of monotone increasing or monotone decreasing, then it is called $\tilde{A}$ is a IFN increasing matrix or a IFN decreasing matrix, where $i=1,2,\cdots,m;\ j=1,2,\cdots,n$.

Definition 7. Let $\tilde{A}_{m\times n}=(\tilde{a}_{ij})_{m\times n}$ be a IFN monotone matrix, where $\tilde{a}_{ij}=<\tilde{\alpha}_{ij},\tilde{\beta}_{ij}>$. If $\overline{A}_{m\times n}=(\overline{a}_{ij})_{m\times n}$, where $\underline{a}_{ij}=<\underline{\alpha}_{ij},\underline{\beta}_{ij}>$, with $<\mu_{\overline{a}_{ij}},\nu_{\overline{a}_{ij}}>$, which is meaning of the maximum degree of membership and minimum degree of membership , and $\overline{a}_{ij}=\frac{1}{2}\int_{-1}^{1}(f_{ij}(x)+g_{ij}(x))(1-x)dx$, then it is called $\overline{A}_{m\times n}$ is a projective matrix of $\tilde{A}_{m\times n}$.

Definition 8. Let $\tilde{A}_{m\times n}=(\tilde{a}_{ij})_{m\times n}$ and $\tilde{B}_{m\times n}=(\tilde{b}_{ij})_{m\times n}$ are two monotone matrixes, where $\tilde{a}_{ij}=<\tilde{\alpha}_{ij},\tilde{\beta}_{ij}>$ and $\tilde{b}_{ij}=<\tilde{\gamma}_{ij},\tilde{\vartheta}_{ij}>$. If $\tilde{\alpha}_{ij},\tilde{\gamma}_{ij}$ are the same sequence, and $\tilde{\beta}_{ij},\tilde{\vartheta}_{ij}$ are the same sequence, then it is called $\tilde{A}_{m\times n}$ and $\tilde{B}_{m\times n}$ are the same sequence of and monotone matrix of IFN.

Definition 9. Assume the four tuple $G=(S^m,S^n,\tilde{A}_{m\times n},\tilde{B}_{m\times n})$ be a IFN matrix of same sequencing and monotone with the payoffs of IFN, with $\tilde{A}_{m\times n}$ and $\tilde{B}_{m\times n}$ which are the same sequence of and monotone matrix on IF-set, it is called the pair of $(x^*,y^*)\in(S^m,S^n)$ is a rational strategy of double matrix games on IF-set. If $x^{\mathrm{T}}\tilde{A}_{m\times n}y^*\le x^{*\mathrm{T}}\tilde{A}_{m\times n}y^*,\forall x\in S^m,x\ge 0^n,x^{*\mathrm{T}}\tilde{B}_{m\times n}y\le x^{*\mathrm{T}}\tilde{B}_{m\times n}y^*,\forall y\in S^n,y\ge 0^n$, then it is called $x^{*\mathrm{T}}\tilde{A}_{m\times n}y^*$ and $x^{*\mathrm{T}}\tilde{B}_{m\times n}y^*$ are the values of Player I and Player II on $G=(S^m,S^n,\tilde{A}_{m\times n},\tilde{B}_{m\times n})$. It is called $(x^*,y^*)\in(S^m,S^n)$ is an optimum strategy of $G=(S^m,S^n,\tilde{A}_{m\times n},\tilde{B}_{m\times n})$.

Theorem 4 Let $\underline{G}=(S^m,S^n,\overline{A}_{m\times n},\overline{B}_{m\times n})$ be a classical and double matrix game, $G=(S^m,S^n,\tilde{A}_{m\times n},\tilde{B}_{m\times n})$ be a IFN and double matrix game. If $\overline{A}_{m\times n}$ and $\overline{B}_{m\times n}$ are the projective matrixes of IFN, then it is easily seen that $\tilde{G}$ and $\overline{G}$ have the same optimum strategy.

Prove.

Let $\tilde{A}_{m\times n}=(\tilde{a}_{ij})_{m\times n}$, where $\tilde{a}_{ij}=<\tilde{\alpha}_{ij},\tilde{\beta}_{ij}>$, and $\tilde{\alpha}_{ij}=f_{ij}(E),\tilde{\beta}_{ij}^{C}=g_{ij}(E)$. According to Definition 9, $\underset{\sim}{A}_{m\times n}$ and $\underset{\sim}{B}_{m\times n}$ are two same sequence of IFN matrixes. Without loss of generality, assume that they be a IFN matrix of same sequencing and monotone increasing. As for $\tilde{A}_{m\times n}$, $f_{ij}(x)$ and $g_{ij}(x)$ are monotone increasing.

Again,

$$x^{\mathrm{T}}\tilde{A}_{m\times n}y^{*}=(x_1,\wedge,x_m)<f_{ij}(E),[g_{ij}(E)]^{C}>_{m\times n}(y_1^{*},\wedge,y_n^{*})^{\mathrm{T}}=\sum_{i=1}^{m}\sum_{j=1}^{n}x_iy_j^{*}<f_{ij}(E),[g_{ij}(E)]^{C}>,$$

$$=<f_1(E),[g_1(E)]^{C}>$$

$x^{*\mathrm{T}}\tilde{A}_{m\times n}y^{*}=\sum_{i=1}^{m}\sum_{j=1}^{n}x_i^{*}y_j^{*}<f_{ij}(E),[g_{ij}(E)]^{C}>=<f_2(E),[g_2(E)]^{C}>$, because of $x_iy_j^{*}\ge 0,x_i^{*}y_j^{*}\ge 0$, it is easily seen that $<f_{ij}(E),[g_{ij}(E)]^{C}>$ is a IFN matrix of same sequencing and monotone increasing, where $(x_iy_j^{*}<f_{ij}(E),[g_{ij}(E)]^{C}>),(x_i^{*}y_j^{*}<f_{ij}(E),[g_{ij}(E)]^{C}>)$.

According to Theorem 2, $(<f_1(E),[g_1(E)]^{C}>)=\{\sum_{i=1}^{m}\sum_{j=1}^{n}x_iy_j^{*}<f_{ij}(E),[g_{ij}(E)]^{C}\}(E)$, $(<f_2(E),[g_2(E)]^{C}>)=\{\sum_{i=1}^{m}\sum_{j=1}^{n}x_i^{*}y_j^{*}<f_{ij}(E),[g_{ij}(E)]^{C}>\}(E)$, then it is easily seen that $<f_1(E),[g_1(E)]^{C}>$ and $<f_2(E),[g_2(E)]^{C}>$ are the same sequence.

Then according to definition 5,

$$x^{\mathrm{T}}\tilde{A}_{m\times n}y^{*}\le x^{*\mathrm{T}}\tilde{A}_{m\times n}y^{*}\Leftrightarrow\frac{1}{2}\int_{-1}^{1}(f_2(x)+[g_2(x)]^{C})(1-x)dx\le\frac{1}{2}\int_{-1}^{1}(f_1(x)+[g_1(x)]^{C})(1-x)dx,$$

$$\int_{-1}^{1}(f_2(x)+[g_2(x)]^{C})(1-x)dx=\int_{-1}^{1}\sum_{i=1}^{m}\sum_{j=1}^{n}x_iy_j^{*}<f_{ij}(E),[g_{ij}(E)]^{C}>(1-x)dx$$

$$=\sum_{i=1}^{m}\sum_{j=1}^{n}x_iy_j^{*}\int_{-1}^{1}<f_{ij}(E),[g_{ij}(E)]^{C}>(1-x)dx=2x^{\mathrm{T}}\overline{A}y^{*},$$

$$\int_{-1}^{1}(f_1(x)+[g_1(x)]^{C})(1-x)dx=\int_{-1}^{1}\sum_{i=1}^{m}\sum_{j=1}^{n}x_i^{*}y_j^{*}<f_{ij}(E),[g_{ij}(E)]^{C}>(1-x)dx=$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n}x_i^{*}y_j^{*}\int_{-1}^{1}<f_{ij}(E),[g_{ij}(E)]^{C}>(1-x)dx=2x^{*\mathrm{T}}\overline{A}y^{*},$$

$$x^{\mathrm{T}}\tilde{A}y^{*}\le x^{*\mathrm{T}}\tilde{A}y^{*}\Leftrightarrow x^{\mathrm{T}}\overline{A}y^{*}\le x^{*\mathrm{T}}\overline{A}y^{*}$$

Empathy may permit, $x^{*\mathrm{T}}\tilde{B}_{m\times n}y\le x^{*\mathrm{T}}\tilde{B}_{m\times n}y^{*}\Leftrightarrow x^{*\mathrm{T}}\overline{B}_{m\times n}y\le x^{*\mathrm{T}}\overline{B}_{m\times n}y^{*}$.

Then according to Definition 8 and the solution expression of classical and double matrix games , it is easily seen that $\tilde{G}=(S^{m},S^{n},\tilde{A}_{m\times n},\tilde{B}_{m\times n})$ and $\overline{G}=(S^{m},S^{n},\overline{A}_{m\times n},\overline{B}_{m\times n})$ have the same values.

In conclusion, the solution of IFN matrix games are calculated as follows:

(i) Generate $\tilde{A}_{m\times n}$ and $\tilde{B}_{m\times n}$, which are two IFN matrixes of same sequencing and monotone, based on fuzzy structured element;

(ii) Obtain the projected matrix according to $\tilde{A}_{m\times n}$ and $\tilde{B}_{m\times n}$;

(iii) Calculate the solution (x^{*},y^{*}) of $\overline{G}=(S^{m},S^{n},\overline{A}_{m\times n},\overline{B}_{m\times n})$;

(iv) Obtain the fuzzy profits of Player I and Player II, if only let (x^{*},y^{*}) be into $x^{*\mathrm{T}}\tilde{A}_{m\times n}y^{*}$ and $x^{*\mathrm{T}}\tilde{B}_{m\times n}y^{*}$.

5 An application

According to the example of literature [20], suppose that there are two companies A and B, aiming to enhance the market share of a product under the same industry. If the market share of A increases, B will decrease, and vice versa. Then the problem of the two companies may be seen a matrix game. Namely, A and B are regarded as Player I and II, and each Player use 4 kinds of strategies.

Let $\tilde{G}=(S^m,S^n,\tilde{A}_{m\times n},\tilde{B}_{m\times n})$ be the zero-sum matrix games of Player I and I, with payoffs of is TIFN generated by Definition 6. Then the mathematical model can be obtained as follows:

$$\tilde{A}_{4\times 4}=\begin{bmatrix}0 & 0.3 & 0.7 & 0.8\\ 0.7 & 0 & 0.5 & 0.7\\ 0.3 & 0.5 & 0 & 0.7\\ 0.2 & 0.3 & 0.3 & 0\end{bmatrix},\quad \tilde{B}_{4\times 4}=\begin{bmatrix}0 & -0.3 & -0.7 & -0.8\\ -0.7 & 0 & -0.5 & -0.7\\ -0.3 & -0.5 & 0 & -0.7\\ -0.2 & -0.3 & -0.3 & 0\end{bmatrix}.$$

Where each element of $\tilde{A}_{4\times 4}$ is denoted as follows:

$0=<(0;0,0),[(0;0,0)]^C>$, $0.3=<(0.3;0.2,0.2),[(0.3;0.25,0.25)]^C>$, $0.7=<(0.7;0.2,0.2),[(0.7;0.22,0.22)]^C>$, $0.8=<(0.8;0.1,0.1),[(0.8;0.15,0.15)]^C>$, $0.7=<(0.7;0.2,0.2),[(0.7;0.25,0.25)]^C>$, $0=<(0;0,0),[(0;0,0)]^C>$, $0.5=<(0.5;0.2,0.2),[(0.5;0.25,0.25)]^C>$, $0.7=<(0.7;0.2,0.2),[(0.7;0.23,0.23)]^C>$, $0.3=<(0.3;0.2,0.2),[(0.3;0.22,0.22)]^C>$, $0.5=<(0.5;0.2,0.2),[(0.5;0.25,0.25)]^C>$, $0=<(0;0,0),[(0;0,0)]^C>$, $0.7=<(0.7;0.2,0.2),[(0.7;0.28,0.28)]^C>$, $0.2=<(0.2;0.1,0.1),[(0.2;0.15,0.15)]^C>$, $0.3=<(0.3;0.2,0.2),[(0.3;0.23,0.23)]^C>$, $0=<(0;0,0),[(0;0,0)]^C>$.

Each element of $\tilde{B}_{4\times 4}$ is denoted as follows:

$0=<(0;0,0),[(0;0,0)]^C>$, $-0.3=<(-0.3;-0.2,-0.2),[(-0.3;-0.25,-0.25)]^C>$, $-0.7=<(-0.7;-0.2,-0.2),[(-0.7;-0.22,-0.22)]^C>$, $-0.8=<(-0.8;-0.1,-0.1),[(-0.8;-0.15,-0.15)]^C>$, $-0.7=<(-0.7;-0.2,-0.2),[(-0.7;-0.25,-0.25)]^C>$, $0=<(0;0,0),[(0;0,0)]^C>$, $-0.5=<(-0.5;-0.2,-0.2),[(-0.5;-0.25,-0.25)]^C>$, $-0.7=<(-0.7;-0.2,-0.2),[(-0.7;-0.23,-0.23)]^C>$, $-0.3=<(-0.3;-0.2,-0.2),[(-0.3;-0.22,-0.22)]^C>$, $-0.5=<(-0.5;-0.2,-0.2),[(-0.5;-0.25,-0.25)]^C>$, $0=<(0;0,0),[(0;0,0)]^C>$, $-0.7=<(-0.7;-0.2,-0.2),[(-0.7;-0.28,-0.28)]^C>$, $-0.2=<(-0.2;-0.1,-0.1),[(-0.2;-0.15,-0.15)]^C>$, $-0.3=<(-0.3;-0.2,-0.2),[(-0.3;-0.23,-0.23)]^C>$, $0=<(0;0,0),[(0;0,0)]^C>$,solving the quasi optimum strategy of $\tilde{G}$.

Solving :

(i) According to the Theorem 4, $\tilde{A}_{4\times 4}=(<f_{ij}(E),[g_{ij}(E)]^{\mathrm{C}}>)_{4\times 4},\tilde{B}_{4\times 4}=(<h_{ij}(E),[l_{ij}(E)]^{\mathrm{C}}>)_{4\times 4}$, with $f_{ij}(E),h_{ij}(E)$, which are the same sequence of and monotone functions and $g_{ij}(E),l_{ij}(E)$ is the same.

then, $f_{11}(E)=0$, $[g_{11}(E)]^C=0$, $f_{12}(E)=0.3+0.2E$, $[g_{12}(E)]^C=0.3+0.25E$, $f_{13}(E)=0.7+0.2E$, $[g_{13}(E)]^C=0.7+0.22E$, $f_{14}(E)=0.8+0.1E$, $[g_{14}(E)]^C=0.8+0.15E$, $f_{21}(E)=0.7+0.2E$, $[g_{21}(E)]^C=0.7+0.25E$, $f_{22}(E)=0$, $[g_{22}(E)]^C=0$, $f_{23}(E)=0.5+0.2E$, $[g_{23}(E)]^C=0.5+0.25E$, $f_{24}(E)=0.7+0.2E$, $[g_{24}(E)]^C=0.7+0.23E$, $f_{31}(E)=0.3+0.2E$, $[g_{31}(E)]^C=0.3+0.22E$, $f_{32}(E)=0.5+0.2E$, $[g_{32}(E)]^C=0.5+0.25E$, $f_{32}(E)=0.5+0.2E$, $[g_{32}(E)]^C=0.5+0.25E$, $f_{33}(E)=0$, $[g_{33}(E)]^C=0$, $f_{34}(E)=0.7+0.2E$, $[g_{34}(E)]^C=0.7+0.28E$, $f_{41}(E)=0.2+0.1E$, $[g_{41}(E)]^C=0.2+0.1E$, $f_{42}(E)=0.3+0.2E$, $[g_{42}(E)]^C=0.3+0.23E$, $f_{43}(E)=0.3+0.2E$, $[g_{43}(E)]^C=0.3+0.28E$, $f_{44}(E)=0$,

$[g_{44}(E)]^C = 0$.

(ii) Generate $\overline{A}_{4\times4} = \begin{bmatrix} 0 & \frac{51}{60} & \frac{376}{300} & \frac{91}{60} \\ \frac{5}{4} & 0 & \frac{51}{60} & \frac{377}{300} \\ \frac{69}{150} & \frac{51}{60} & 0 & \frac{93}{75} \\ \frac{19}{60} & \frac{137}{300} & \frac{11}{25} & 0 \end{bmatrix}$, which is a projective matrix of IFN, and according to zero-sum, it is clearly seen as following: $\overline{B}_{4\times4} = -\overline{A}_{4\times4}$.

(iii) Solving the equation of $\overline{G} = (S^m, S^n, \overline{A}_{m\times n}, \overline{B}_{m\times n})$, and obtaining the solution of (x^*, y^*), that is $x^* = (0.2598, 0.2473, 0.4928, 0.0001), y^* = (0.2578, 0.4909, 0.2512, 0.0001)$

(iv)Then the profits of A team and B team are seen as follows: $Z_{\tilde{A}} = x^{*T}\tilde{A}_{m\times n}y^* =< (0.5869; 0.4564, 0.4564), [(0.5869; 0.4913, 0.4913)]^C >$, $Z_{\tilde{B}} = x^{*\mathrm{T}}\tilde{B}_{m\times n}y^* =< (-0.5869; 0.4564, 0.4564), [(-0.5869; 0.4913, 0.4913)]^C >$.

Then the degrees of membership and non-membership are seen as follows:

$$\mu_{\tilde{A}}(x) = \begin{cases} 1 + \dfrac{x-0.5869}{0.4564}, 0.1305 \le x \le 0.5869; \\ 1 - \dfrac{x-0.5869}{0.4564}, 0.5869 < x \le 1.0433; \\ 0, \text{otherwise}. \end{cases} \quad \nu_{\tilde{A}}(x) = \begin{cases} -\dfrac{x-0.5869}{0.4913}, 0.0956 \le x \le 0.5869; \\ \dfrac{x-0.5869}{0.4913}, 0.5869 < x \le 1.0782; \\ 0, \text{otherwise}. \end{cases}$$

6 Conclusion

We mainly researches the solution of Nash equilibrium about IF matrix games with the payoffs of IFN, based on fuzzy structured element. The comparison of IFN is transformed into the comparison of monotone functions, then the original problem is equally transformed into the classical problem of matrix games with payoffs of precise numbers. By comparison, the advantages are clearly seen as follows:

(i) the sequence based on IFN structured order is better than general orders, and satisfy the rational hypothesis of the economic man;

(ii) it can be applied to any problem of matrix games with payoffs of IFN, namely, only one method is established without need for specific forms of IFN.

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