Analysis of Optimal Beta Value Calculation and Index Futures Trading

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Abstract: This paper mainly analyzes the relationship between optimal beta value and stock index futures trading. With the total value of the total investment determined, calculate the beta value of the optimal portfolio, and analyze how to preserve the optimal investment, how to change the value-saving effect if the Shanghai index rises, how to change the value-preserving effect if the Shanghai index falls, and summarizes the main role and trading advantages of stock index futures.

1. Introduction

Stock beta values indicate how sensitive the portfolio is to systemic risk: A beta value of 1 indicates that when the index changes, the stock price changes at the same percentage for the average risk stock; Beta value smaller than 1 for low-risk stocks, beta value greater than 1, for high-risk stocks. In general, what true meaning is a measure of the systemic risk of a particular asset. Share Price Index Futures refer to standardized futures contracts based on stock price indices, and both parties agree that on a specific future date, they can trade and sell the underlying index according to the size of the pre-determined stock price index, and settle the difference through cash after maturity.

Based on the sample of six stocks on the Shanghai Stock Exchange, the trading data of 2019.01.02-2019.5.19 were selected to calculate the beta value of each stock. Assuming that the current Shanghai index is 3400 points, the total market value of the current investment is 6.8 million yuan, six months later the index points of the Shanghai Index futures contract is 3450. Details of the selected shares are as follows:

Dongbai Group is a department store retail-based integrated company. In 2018, the Group's net income was RMB262 million, up 5.89% YoY, and its operating income was RMB2,997 million, up -22.27% YoY. In order to select the best stock portfolio, we also selected five other stocks on the Shanghai Stock Exchange, which are similar to the Dongbai Group. They are: Central Mall, Eurasia Group, Han Shang Group, Chongqing Department Store, Xiamen International Trade. All five are in the retail sector of commercial department stores and are similar to Dongbai Group in their corporate operations. Table 1 collects the share prices of the above-mentioned groups for 20 weeks during the trading period.

Nanjing Central Mall Co., Ltd, the main department store, warehouse supermarket and real estate industry. The net profit margin for the current period was 3.11%, compared with 3.37% in the same period last year, and the operating efficiency of the enterprise decreased, with the net profit margin of the current period being 0.57%, the total asset income capacity of 0.56%. in the same period last year, and the return on net assets of 1.99% in the current period, compared with 1.69% in the same period last year, and the enhancement of the ability of return shareholders.

Chongqing Department Store Co Ltd. is mainly involved in department stores, supermarkets, electrical appliances and auto trade and other business areas.

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Table1 the beta value of each stock.

Stock code	Stock Name	1	2	3	4	5	6	7	8	9	10
600693	Dongbai Group	5.3	5.26	5.23	5.21	5.28	5.32	5.37	5.62	5.98	6.37
600280	Central Mall	3.56	3.49	3.85	4.54	3.67	4.34	4.21	4.53	4.65	4.6
600697	Eurasia Group	18.69	18.57	18.84	18.86	18.73	19.29	19.85	20.68	20.82	21.02
600774	Han Shang Group	13.35	12.44	11.75	11.46	10.72	11.41	12.4	12.77	13.33	14.27
600729	Chongqing Department Store	28.01	27.8	27.95	27.82	28.4	28.92	29.37	30.85	31.99	34.69
600755	Xiamen International Trade	7.18	7.08	7.08	6.98	7.04	7.21	7.72	8.02	8.17	8.6
Stock code	Stock Name	11	12	13	14	15	16	17	18	19	20
600693	Dongbai Group	6.08	6	6.11	6.1	6.08	5.75	5.6	5.26	5.34	5.35
600280	Central Mall	4.73	4.46	4.71	5.12	4.92	4.41	4.31	4.07	4.08	3.95
600697	Eurasia Group	21.98	22.19	23.21	21.63	20.78	19.75	19.87	19.08	18.85	18.89
600774	Han Shang Group	14.15	13.63	14.23	13.61	14.16	13.19	12.71	12.3	12.88	12.2
600729	Chongqing Department Store	35.9	37.18	38.65	37.95	35.66	32.52	34.9	31.61	31.64	30.62
600755	Xiamen International Trade	9.56	9.19	9.66	9.17	9.21	8.54	8.81	8.83	8.69	8.13

Xiamen International Trade Group Co., Ltd. is a large-scale integrated enterprises. The return on net assets after the 2018 report was 4.23%, compared with 7.12% in the same period last year, and the contribution of the main business profit decreased significantly.

Wuhan Han Shang Group Co., Ltd. is a retail, exhibition, foreign trade, tourism in one of the commercial listed companies. Gross margin of 23.36%, compared with 22.94% in the same period last year, the main profitability remained stable, net profit margin was 3.11%, compared with 3.37% in the same period last year, business efficiency decreased.

2. Build a Model

$$\min \omega^T V \omega \tag{1}$$

$$s.t \sum_{i=1}^{n} x_i = 1 \tag{2}$$

$$\omega = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} = \frac{1}{D}(BV\vec{1} - AV^{-1}e) \tag{3}$$

$$A = \vec{1}V^{-1}e \tag{4}$$

$$B = e^t V^{-1} e \tag{5}$$

$$C = \vec{1}V^{-1}\vec{1} \tag{6}$$

$$D = BC - A^2 \tag{7}$$

$$\vec{1} = \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix} \tag{8}$$

$$\vec{e} = \begin{pmatrix} \overline{r_1} \\ \overline{r_2} \\ \vdots \\ \overline{r_n} \end{pmatrix} \tag{9}$$

By calculating the matrix, the results are as follows (10)

$$\omega = \begin{pmatrix} 0.958905935 \\ -0.011618727 \\ 0.082623796 \\ -0.032167334 \\ -0.098620234 \\ 0.068045812 \end{pmatrix} \tag{10}$$

Similarly, through the formula (11)

$$\beta_i = \frac{Cov(r_i, r_m)}{Var(r_m)} \tag{11}$$

We can calculate the beta coefficient for the six stocks.

Table 2 beta coefficient

β1	β2	β3	β4	β5	β6
0.173	0.198	0.371	0.078	1.05	2.164

The investment ratio factor derived using the above formula has a negative value, it means that the i asset securities should be sold rather than bought. But in China's stock market. Therefore, we need to adjust the model to make the quality investment ratio coefficient non-negative.

Therefore, the following is to use projection algorithm to solve the relevant investment scale coefficient. Set the following linear inequality

$$\begin{cases} a_i x = b_i, i = 1, \cdots e_i \\ a_i x \succ b_i, i = e+1, \cdots, m \end{cases} \tag{12}$$

Where a_i is n-dimensional line vector,i=1,…..,m, $b_1,\ldots,b_m$ is a real number,x is n-dimensional column vector,0≤e≤m.The projection algorithms for linear inequality groups are related to the following concepts:

Coefficient vectors $a_1,\ldots,a_m$,One of the largest linear unrelated groups is called the base vector group, the vector in the base vector group is called the base vector, and the remaining vectors are called non-base vectors.

（2）The corresponding (not) equation of the base vector is called the base (not) equation, and the corresponding (not) equation of the non-base vector is called the non-base (non) equation.

（3）The solution set of inequality groups consisting of base equations and base inequalities is called the base cone.

xL is the vertex of the current base cone, and if x^L does not satisfy an inequality (called a violation of inequality), the inequality is exchanged with an inequality in the base cone to obtain a new base cone, and the vertices of the new base cone meet the base inequality. Repeat the process until a solution of the inequality group is obtained or found unsolvable.

I0 and I1 are the number sets of base equations and base inequalities,I2 is the number set of non-base inequalities, and the vertices of the base cones are x^L. The deviation of the $\sigma_i=a_ix^L-b_i$ called the deviation of the a_i or the corresponding (not) equation.

Set up a new

$$a_i = \sum_{j\in I_0\cup I} \omega_{ij} a_j, \quad i \in I_2 \tag{13}$$

If $\sigma_r = a_r x^L - b_r < 0 (r \in I2)$,then $a_r x \geq b$ are available as a base inequality. For all $j \in I_1$.

If $\omega_{rj} \leq 0$, there is no solution. Otherwise there is $\omega_{rs} > 0 (s \in I_1)$, The corresponding inequality $a_sx \geq b_s$ is available as a base inequality. The number set of the violation inequality is $\bar{I}$, $\bar{I} = \{i\text{:} \ \ \sigma_i = a_i x^L - b_i < 0, i \in I_2\}$

If $\sigma_k = min\{\sigma_i : i \in \bar{I}\}$, and $\omega_{rs} > 0, s \in \bar{I}$.

If $$\frac{-\sigma}{\omega_{rs}} = max\{\frac{-\sigma}{\omega_{rs}} : \omega_{rs} \succ 0, S \in \overline{I}\} \tag{14}$$

Then with $a_r x \geq b_r$ into the base with $a_s x \geq b_s$ out of base, The aim is to make the more inequality deviation of the more violation situ into non-negative.

The process and results are as follows:

$$\begin{cases} min\omega^T V \omega \\ s.t \sum_{i=1}^{n} X_i = 1 \end{cases} \tag{15}$$

V meets the positive condition, the problem can be optimally solved. Using the Kuhn-Tucker condition to get:

$1.20\omega1+ 0.66\omega2+ 1.62\omega3+ 2.853\omega4+ 6.898\omega5+ 0.615\omega6-\lambda1=0$
$0.66\omega1+5.46\omega2+2.85\omega3+2.3837\omega4+5.4094\omega5+1.1245\omega6-\lambda2=0$
$1.61\omega1+2.84\omega2+17.07\omega3+6.67\omega4+28.73\omega5+7.10\omega6-\lambda3=0$
$2.85\omega1+2.38\omega2+6.67\omega3+17.35\omega4+15.80\omega5+5.49\omega6-\lambda4=0$
$6.90\omega1+5.41\omega2+ 28.73\omega3+15.80\omega4+99.75\omega5+13.70\omega6-\lambda5=0$
$0.61\omega1+1.12\omega2+7.10\omega_3+5.49\omega_4+13.70\omega_5+6.22\omega_6-\lambda_6=0$

$$\sum_{i=1}^{n} \omega_i = 1 \tag{16}$$

$$\omega_i \geq 0, \quad \lambda_i \geq 0 ? = 1,2,3,4,5,6$$

With $\omega i \geq 0, \lambda i \geq 0$ as the initial base cone, the vertex of the base cone is $\omega_j=0$,j=1,2,3,4,5,6; $\lambda_j=0$,j=1,2,3,4,5,6.The initial base vector is e_j, j=1,2,…,7.ej is the 7-bit unit line vector of the j component 1.Set 7 equations with coefficient vectors area1,…,a7.

Their deviations are:

σ_1=-1.61M,σ_2=-1.12M, σ_3=-1.10M,σ_4=-5.49M,σ_5=-1.69M,σ_6=-2.22M,σ_7=-1.

(M is a sufficiently large positive number)

Table 3 The initial projection table method

	e1	e2	e3	e4	e5	e6	e7	e8	e9	e10	e11	e12	σi
a1	1.2	0.66	1.62	2.85	6.9	0.61	-1						-1.16M
a2	0.66	5.46	2.85	2.38	5.41	1.12		-1					-1.12M
a3	1.62	2.85	17.1	6.67	28.7	7.1			-1				-1.1M
a4	2.85	2.38	6.67	17.4	15.8	5.49				-1			-5.49M
a5	6.9	5.41	28.7	15.8	99.8	13.7					-1		-1.69M
a6	0.61	1.12	7.1	5.49	13.7	6.22						-1	-2.22M
a7	1	1	1	1	1	1							-1

In order to facilitate the calculation, as far as possible to 1 as a hub, according to the above method to determine the base inequality.

First, determination of the in-base variable. From the optimal interpretation of the theorem can be seen, when a certainδi>0, the non-base variable xi does not take zero value can make the target function value increase, so we have to choose the base test number greater than zero non-base variable to the base variable. If there are more than two, the non-base variable of the largest δi>0is the invariant variable.

Second, the determination of the out-base variable. The coefficients of the established inbound variables in each constraint equation are divided by the values of the constant term in the constraint equation in which they are located, and the base variables in the constraint equation in which the smallest ratio is located are determined as the base variables. The optimality is then tested, and if it is not the optimal solution, the base change is to continue until the optimal solution is found.

In Table 4, first with a1 into the base, e_7 out of the base after iteration and calculation, get the following table

Table 4 The coefficients of the established inbound variables

	e_1	e_2	e_3	e_5	e_6	e_9	e_{10}	e_{11}	e_{12}	σi
a_3	8.92	6.89	57.5	32.7	6.1	-2				3.56
e_1	9.88	5.78	6.63	36.8	15.8		-1			0.51
e_2	6.53									0.23
e_3	6.65									0.15
e_4	4.35									0.05
e_5	7.32	5.41	28.7	99.8	13.7			-1		0.03
e_6	5.76	1.12	7.1	13.7	6.22				-1	0.02

Through related calculations and iterations, you get:

$e_1=0.511, e_2=0.231, e_3=0.156, e_4=0.045, e_5=0.034, e_6=0.023$.

The corresponding optimal scenarios are: the percentage of funds occupied by the six securities is 51.1%, 23.1%, 15.6%, 4.5%, 3.4%, 2.3%.

The combination of beta values:

$$\beta_I = \sum_{i=1}^{n} x_i \beta_i = 0,325 \tag{17}$$

3. Analysis of Hedging Strategies

Known:$\beta i=3.828. I_p=680, p^0=3400, p^t=3450, m=300$yuan/point,

The number of copies of the SSE futures contract sold is:

$$N = \frac{I_p}{m * p^0} * \beta_1 = \frac{6800000}{300 * 3400} * 0.20285 = 2(copies) \tag{18}$$

During this period, the Shanghai Composite Index rose（3450-3400）/3400=1.471%, The value of the equity portfolio changes to

$\Delta I_p=1.471\%*0.325863*6800000=32595.4$(RMB)

The loss resulting from the sale of SSE futures is

N*m*（3450-3400）=-2*300*50=-30000

Overall hedging effect: 32595.4 -30000=2595.42(RMB)

If the index falls to 3200 in December or the index falls to 3500 in December, the results are follows

Table5 The result of different index

index	3200.00	3500.00
the Shanghai Composite Index	-0.07	1.449%,
The value of the equity portfolio（RMB）	-162501.30	32022.90
The gain/loss from selling SSE futures	123000.00	-300000.00
Overall hedging effect（RMB）	-39501.30	-267977.10

4. General Conclusion:

The main role of stock index futures

First, stock index futures have the function of risk aversion. By calculating the beta value and the relevant data, we can allocate the investment according to the current stock market situation, reasonably avoid risk, and choose the best portfolio. This is achieved through hedging, investors can be in the stock market and stock index futures market two market reverse operation to achieve the purpose of risk aversion, is conducive to investors to avoid the stock spot market price risk.

Second, stock index futures have the function of price discovery. Because of the large number of participants in stock index futures, and different participants can participate in stock index futures quotation. According to their own expectations, the former stock index futures price naturally contains all aspects of price expectations information. Besides, the two-way trading mechanism of stock index futures makes market pricing more rational.

Finally, stock index futures have asset allocation function. The subject matter of stock index futures is the stock index, and its reasonable cash delivery system ensures that the stock index futures are highly correlated with the trend of the underlying stock index, which makes stock index futures have the function of asset allocation.

Trading Advantage of Stock Index Futures

Depending on market trends, additional returns can be obtained by selecting securities with different beta factors. Can buy up or down？ If the price fell down can sell, the price up can do more. As long as the market trend is correctly estimated, ups and downs can make money. In addition, stock index futures have leverage and will make more profitable if the right choice is made.

The beta factor reflects the sensitivity of securities to the market, in a very sure forecast to the arrival of the bull market, should choose those high beta coefficient of securities, it will multiply the market yield, resulting in high returns.

To avoid non-systemic risk, securities with the same beta coefficient can be selected for portfolio investment under the corresponding market trend. Investors with strong psychological tolerance choose aggressive portfolio strategy, psychological affordability is weak choice conservative portfolio strategy, and investors choose neutral portfolio strategy between the two.

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