SVGLRAM---A New Type of Non-International Algorithm to Achieve Image Compression

YunPeng Fan

Basic Department of Xi'an Siyuan University, Xi an, Shan xi China

478235181@qq.com

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Abstract: SVGLRAM-----a new type of non-international algorithm to achieve image compression, with its advantage of the acquisition of minimum error value, the closer compression ratio and the comparatively lower time and space computational complexity for the purpose of avoiding the weaknesses of SVD and GLRAM, and it has been experimented in different facial databases such as ORL and Yale with its high efficiency.

1. Main Methods of Image Compression

Image compression is to save storage space and increase transmission speed. So far, scholars have proposed many methods as follows: (1) PCA algorithm [1]: proposed by Turk and Puntland [2] of MIT Media Lab, its basis is kauhumen-Loeve transformation. The basic principle is: using K-L transform to extract the main components of the face to form the feature face space, projecting the test image to this space during recognition, and obtaining a set of projection coefficients, , Through the comparison with each face image for recognition.(2) 2DPCA algorithm[3,4,5,6]: by Yang[7] et a. In 2004, it was proposed that the principle is as follows: with an image, first calculate the mean value, then calculate the total scatter matrix, find the eigenvalue and eigenvector, and select the eigenvector corresponding to the previous maximum eigenvalue to form the moment, Matrix, calculate the reconstructed image.(3) SVD algorithm: image matrix is decomposed into singular value matrix and orthogonal matrix respectively, which contain the main information of image matrix. The front and column of the sum are recorded as, and the image is reconstructed. When the dimension of matrix is low, the error of SVD decomposition is minimum. When the scale of matrix increases greatly, the calculation of singular value of matrix becomes a difficult problem. (4) GlRAM algorithm [8, 9]: proposed by Ye [10, 11] in 2004, if there is an image, the matrix sum is found by iterative algorithm, and the reconstructed image is used. The algorithm uses bilateral compression, which has a higher compression ratio than SVD, and can compress many images at a time. But its disadvantage is iterative algorithm. After GLRAM, Liang [12], Liu [13], Lu [14] proposed a non-iterative algorithm for calculating sum.

2. New Development of Image Compression Methods

2.1: SVD Algorithm

Image compression is realized by singular value. The data model used is vector space. Any matrix can be decomposed by singular value.

Theorem 1(singular value decomposition theorem)[15,16,17], $A \in R^{m\times n}$: if , $m \ge n$, $rank(A) = r$, then there are two orthogonal matrices, $U = [u_1, u_2 \cdots u_m] \in R^{m\times m}$, $U^T U = I$ and $V = [v_1, v_2 \cdots v_n] \in R^{n\times n}$, $V^T V = I$, and diagonal matrix, $S = diag[\lambda_1, \lambda_2 \cdots, \lambda_r, 0, \ldots 0] \in R^{m\times n}$, $\lambda_1 > \lambda_2 > \cdots > \lambda_r \geqslant 0$, which make the following formula hold:

$$A = U\begin{bmatrix} \Lambda & 0 \\ 0 & 0 \end{bmatrix}V^T = UST^T = \sum_{i=1}^{r} \lambda_i u_i v_i^T \quad \textbf{(1)}$$

Theorem 2: For the SVD representation of truncation error, it is the best approximation in the global sense under Fresenius norm among all the approximations of rank.

2.2 GLRAM

In 2004, YE proposed a new matrix approximation algorithm; generalized low rank approximation of matrix (GLRAM).It uses the matrix space model. The idea is as follows: with training data A_i (i=1,2… n), we need to find two different matrices L $(L \in R^{r\times\ell_1})$, R $(R \in R^{c\times\ell_2})$ and matrices M_i (i=1,2… n), and let the approximation matrix, that is, under the f norm, meet the following formula:

$$\min \sum_{i=1}^{n} \left\| A_i - LM_iR^T \right\|_F^2 \quad \textbf{(2)}$$

Turn matrix approximation into optimization problem. See the following theorem:

Theorem 1: If the matrix L, R and M_i, $i = 1,\cdots n$, optimize the above problem, then for each of them, $M_i = L^T A_i R$, $i = 1\cdots n$

Theorem 2: If the matrix, L, R and M_i, $i = 1,\cdots n$, make the above problem optimal, then the matrix, L, R and M_i, $i = 1,\cdots n$, are equally valid for the optimization problem $\max \sum_{i=1}^{n} \left\| L^T A_i R \right\|_F^2$.

Theorem 3: If the matrix L, R and M_i, $i = 1,\cdots n$, , make the above problem optimal, then there are the following conclusions:

(1) If known R, let

$$M_L = \sum_{i=1}^{n} A_i RR^T A_i^T$$

Then the eigenvectors corresponding to the first ℓ_1 largest eigenvalue of M_L constitute the matrix L.

(2) If known L, let

$$M_R = \sum_{i=1}^{n} A_i^T LL^T A_i$$

Then the eigenvectors corresponding to the first ℓ_2 largest eigenvalue of M_R constitute the matrix R.

According to the above three theorems, the iterative algorithm of GLRAM algorithm for matrix sum is obtained as follows:

GLRAM algorithm

Input:, A_i, $i = 1,\cdots n$, ℓ_1, ℓ_2

Output: L, R, M_i, $i = 1,\cdots n$

One: Give the most primitive $L_0 = \begin{pmatrix} I_{m\times m} \\ 0 \end{pmatrix}$

Two: Calculate the matrix $M_R = \sum_{j=1}^{n} A_j^T L_{i-1} L_{i-1}^T A_j$, then calculate the eigenvectors corresponding to the previous ℓ_2 maximum eigenvalues, and form the matrix R_i with these vectors in columns

Three: Calculate the matrix $M_L = \sum_{j=1}^{n} A_j R_i R_i^T A_j^T$, then calculate the eigenvectors corresponding to the previous ℓ_1 maximum eigenvalues, and form the matrix L_i with these vectors in columns

Four: $i = i + 1$, return to step 2 until the algorithm converges

Five: $L = L_{i-1}$, $R = R_{i-1}$, for (I = 1,, end

2.3 SVGLRAM

In SVD, singular value decomposition of image matrix can get the sum of singular value matrix and orthogonal matrix, and it contains important information of image. Combined with the above two methods, this paper proposes a non-iterative algorithm for summation, called SVGLRAM algorithm. The steps are as follows:

SVGLRAM algorithm

Input: A_i, $i = 1 \cdots n$, k,

Output: L, R, M_i, $i = 1 \cdots n$

One: Calculation $M = \sum_{i=1}^{n} A_i A_i^T$, singular value decomposition, select the front row corresponding matrix as L

Two: Calculation $N = \sum_{i=1}^{n} A_i^T A_i$, singular value decomposition, selects the front row corresponding matrix as R

Three: Calculation: $M_i = \sum_{i=1}^{n} L^T A_i R$, and then reconstruct each image as $\tilde{A}_i = \sum_{i=1}^{n} L M_i R^T$

The experimental environment of this paper is Windows XP; all experiments are completed under the version of matlab2012a. The database used is ORL face database and YALE face database. ORL face database is a series of face images taken by Olivetti laboratory in Cambridge, UK, with 40 objects of different ages, genders and races. Each object consists of 10 images with a total of 400 gray-scale images. YALE face database is also a famous face database, which is made by Yale University computer vision and control center. It includes 15 people, 11 multi pose, and multi illumination images for each person.

2.4 Quality Evaluation

The digital image uses lossy compression, the image before and after compression will produce a certain degree of distortion, from the restoration of image quality, compression comparison of various algorithms for evaluation.

(I) in terms of reduction quality, PSNR is adopted in this paper. Peak signal-to-noise ratio, which is defined as: [18, 19]

$$PSNR = 10\log_{10}\left(\frac{MAX_I^2}{MSE}\right) = 20\log_{10}\left(\frac{MAX_I}{\sqrt{MSE}}\right)$$

Where, represents the maximum value of the image point color. Represents the mean square deviation of two color images I and K,

On the denominator of MSE, the larger the value of MSE, the smaller the value of PSNR, the smaller the value of MSE, the larger the value of PSNR, and the larger the value of PSNR, indicating that the smaller the error between the original image and the reconstructed image, the better the algorithm.

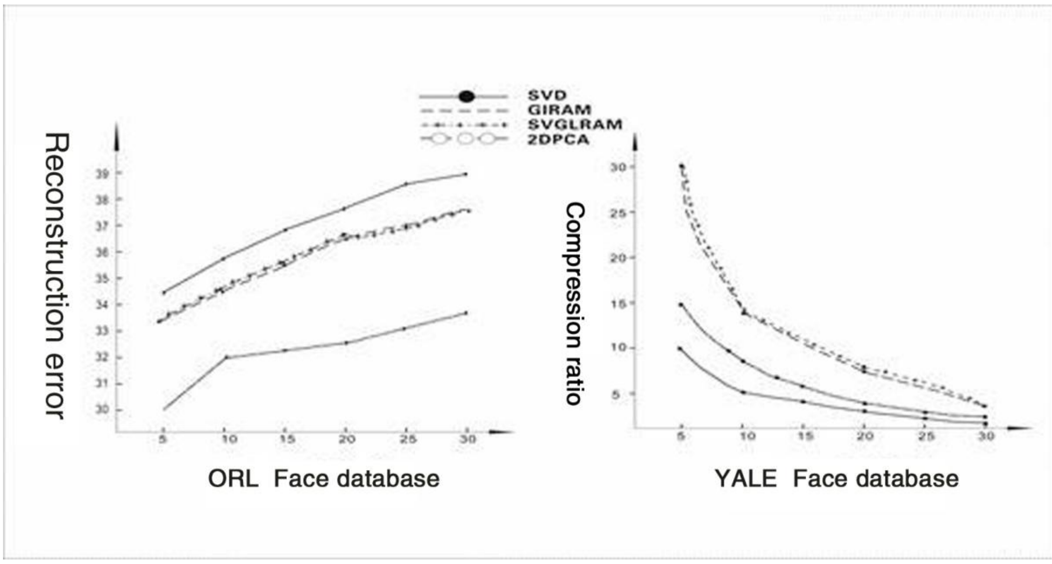

Figure 1 **Figure 2.**

As shown in Figure 1, no matter k = 5, k = 10, k = 20, k = 30, the error of SVD is the smallest. The error of glram and svglram are very close, and the error of 2DPCA is the largest, because it adopts unilateral dimension reduction.

It can be seen from Figure 2 that SVD has the smallest compression rate, followed by 2DPCA. Glram and svglram are very close, with the largest compression rate,

3. Summary and Outlook

Because GLRAM is an iterative method about sum and it can’t prove when and why glram can achieve good experimental results. On the basis of SVD and GLRTAM, this paper proposes a new algorithm of sum. This algorithm is non iterative, and has the same compression ratio and very close error value as GLRAM. How to combine GLRAM algorithm with other algorithms to study optimization problem is a problem that needs to be solved further.

Reference

[1] Yu Linfei, improved pCa-SIFT image matching algorithm, Journal of Xi'an University of engineering, 2020, 34 (2), 3-4

[2] M. Turk, a Pentland. Eigenfaces for Recognition [J].Cogntive Neuroscience J, 1991,3(1):71-86.

[3] Feng Fei, application of improved 2DPCA algorithm in face recognition, computer science, Vol. 44, No. 11a, 2017, 266-269

[4] Yang Meifang, face recognition based on 2DPCA + PCA and SVM, information technology, issue 2, 201832-36

[5] Li Zhinong, research on metal fracture image recognition method based on 2DPCA, Journal of Nanchang University of Aeronautics, Vol. 33, No. 1, 2019, 48-52

[6] Wang Mingxia, face detection research based on an improved 2DPCA algorithm, wireless Internet technology, No. 10, 2018, 137-139

[7]J. Yang, D. Zhang, A. F. Frangi. Two-dimensional PCA: A New Approach to Appearance-Based Face Representation and Recognition 2004, 26(1):131-137.

[8] Peng Yanfei, an improved palmprint recognition algorithm based on glram, computer application and software, No. 10, 2015, 155-158

[9] Sun Yang, an alternative feasibility algorithm for generalized low rank approximation of matrices, Northeast Normal University, 2018.

[10] J. Ye. Generalized Low Rank Approximations of Matrices, Machine learning. 2004, 887-894.

[11] J. Ye. Generalized Low Rank Approximations of Matrices. Machine learning. 2005, 61(13): 167-191.

[12] Z. Liang, P. Shi. An analytical algorithm for generalized low-rank approximationsof matrices. PATTERN RECOGNITION. 2005, 38:2213-2216.

[13] J. Liu, S. Chen. Non-iterative generalized low rank approximations of matrices. Pattern Recognition Letterns. 2006, 27:1002-1008.

[14] C. Lu, W. Liu, S. An. A simplified GLRAM algorithm for face recognition

Neurocomputing. 2008, 72: 212-217

[15] Chen Yaxiong, image compression algorithm based on SVD and contourlet transform, computer application research, Vol. 34, No. 1, 2017, 317-320

[16] Chen Huiyan, simplified data application based on SVD, software guide, Vol. 18, No. 8, 2019, 162-165

[17] Zhao Xuezhi, signal decomposition algorithm based on SVD of variable matrix structure, vibration, test and diagnosis, Vol. 38, No. 6, 2018, 1096-1102

[18] Zhu Liang, an image denoising algorithm based on low rank matrix approximation, Journal of Hangzhou University of Electronic Science and technology, Vol. 35, No. 4, 2015, 36-40

[19] Zhang Xia, classification and evaluation of image compression methods, Journal of Taishan University, Vol. 40, No. 3, 2018, 81-83