Risk Measurement Research Under Behavioral Finance

Sijia Lyu

University of Queensland, Queensland(QLD), Code: 4067

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Abstract: In the last few decades, an increasing amount has been given to risk measurement in the financial market. usually occurred under traditional risk evaluation approach and lead financial market appear unstable trend. In order to solve this problem, this paper will firstly compare with risk measurement under traditional and behavior finance. Based on shortage of behavior finance, this paper will construct new risk measurement method to verify that new risk measurement method is more efficient compared with to research the relationship between risk and return. Finally, this paper will show the disadvantages and suggestions for new risk measurement method.

Introduction

Reducing risk has become a normal phenomenon in modern financial market. However, academia not has theory about diversify risk before 1952 due to classical economists believe that fund as a whole in transaction process and ignore portfolio diversification in investing procession. In 1959, the above situation has changed dramatically because Markowitz theory occurred. Markowitz (1959) utilize variance or standard deviation to evaluate portfolio return and generate mean variance model. Definitely, risk measurement boarded the stage of history and efficient investment portfolio strategies cannot exist independent without risk measurement. Therefore, a great number of economists begin to research risk measurement method.

More evidence show that behavior finance is a subject that combine with finance, psychology, behavioral science, sociology trying to disclose non-rational behavior and decision regulation. It means that internal value is not the factor which has impact on price in security market, it also has largely influence on investor's behavior like the price will change according to psychological alter. In addition, Satia Nur Maharani (2014) pointed the view that behavior finance based on efficient market hypothesis. Assuming investors are rational and could immediately make a reasonable reaction for market information and all valuable information will reflect in stock price trend timely and correctly.

In conclusion, this paper analyses the relationship between return and risk under variance method and new risk measurement approach under financial behavior. In addition, this paper will compare risk evaluation under traditional finance and behavior finance. Next, this paper will apply a new risk measurement method that take psychology factors into consideration to research the relationship between risk and return and illustrated the correlation between two methods. Finally, proposing inadequate, suggestions and prospect at the last chapter in this paper.

2. Risk measurement theory

2.1 Risk measurement under traditional finance

2.1.1Markowitz risk measurement

Before the Markowitz (1959) theory occurred, most of securities agencies prefer to utilize experience and objective judgements to conduct quantitative analysis. Markowitz theory contribute the balance between asset allocation return and risk. It satisfied assumptions that have the maximize return at certain risk level and invertors face the minimize risk under particular return level. Based on these assumptions, Markowitz mean-variance model (mathematical form) can be represented as follows:

$$min \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \ Cov(r_i, r_j)$$

$$\max \sum_{i=1}^{n} \bar{r}_i \, w_i$$

$$Restrictions: \begin{cases} \sum_{i=1}^{n} w_i = 1 \\ \sum_{i=1}^{n} w_i = 1, w_i \geq 0 \end{cases}$$

Based on this formula, the model identify return as abscissa axis and standard deviation as ordinate axis. All of eligible combination form a curve which called efficient frontier. It might achieve the optimistic combination that maximize return and minimize risk under specific weight (w), expected return (r) and risk (σ).

Markowitz theory not only explain determined factors of portfolio assets risk but also illustrate asset return is decided by itself risk. However, due to investors have different risk preference, therefore variance exist positive skewness and negative skewness. As for investor, when the actual return exceeds expected return, investors may not face risk. Investors will have risk when actual return lower than expected return. In conclusion, variance could not reflect return under various risk preferences and may cause distribution has errors.

2.1.2 Coefficient β risk measurement

Coefficient β concept is originated from statistics and it used for measure stock return volatility relative to benchmark return. The amount of β higher means volatility lager.

Financial market including systematic risk and nonsystematic risk. Sharpe (1964) putted forward that using β to measure systematic risk. The mathematical expression is:

$$\beta_i = \frac{R_i - R_f}{R_m - R_f} = \frac{cov(R_m, R_i)}{\sigma_m^2}$$

Based on this formula, R_f and R_m describe risk-free risk and market securities return. $cov(R_m, R_i)$indicate covariance between market portfolio and security i. The Capital Asset Pricing Model (CAPM) is known:

$$R_i = R_f + \beta(R_m - R_f)$$

The formula shows that the security return can be calculated based on risk-free risk, market risk and Coefficient β. The number of Coefficient β could determine the amount and return to measure volatility. It applied widely such as calculate coast of fund thus make investment decisions or make asset valuation.

However, Coefficient β may not comprehensive because it just takes systematic risk into consideration and ignores the influence on nonsystematic risk. More evidence shows that total risk equal systematic risk plus nonsystematic risk. In terms of systematic risk, it cannot be reduced through inclusion of more securities in a portfolio and use Coefficient β to measure it. In theory, nonsystematic risk could diversify with the increase number of portfolios. But in actually, nonsystematic risk cannot equal zero just infinitely close to zero. In conclusion, Coefficient β might has bias. On the other aspects, the changes of low value stock lags behind other stocks and lead errors. Blume (1979) hold the theory that Coefficient β has regression trend and assume that over regression and time are independent in variable. Therefore, Coefficient β can be adjusted and the adjusted formula as follows:

$$\beta_t = a + b\beta_{t-1}$$

The result indicates that β shows a decline trend after adjusted and increase accuracy and reliability.

2.1.3 VaR risk measurement

Based on risk measurement approaches, Markowitz approach may not classify systematic risk and nonsystematic risk clearly and coefficient β method occur bias due to β is unstable. In order to improve the efficient of risk management, Dempster & ProQuest (2002) pointed the view Value at Risk to measure risk. According to quantitative statistics approach, VaR (Value at Risk) could give an expression of potential loss probability under a certain confidence level.

$$\mathrm{P}(\Delta \mathrm{P} \Delta \mathrm{t} \leq VaR) = \alpha$$

In this formula, ΔP illustrate the investment portfolio market value changes during Δt, αmeans confidence level and VaR represent the maximum losses in a certain confidence level and time period.

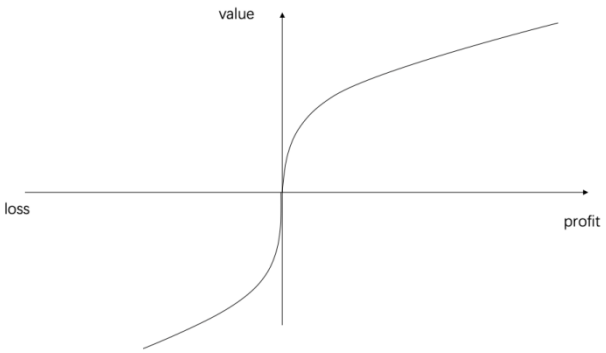
2.2 Risk measurement under behavior finance

2.2.1 Value function and Decision weight function

Generally, personal chooses and decisions can be classified two stages (Kahneman, 1979). One is value function and another is decision weight function. The mathematical theorem as follows:

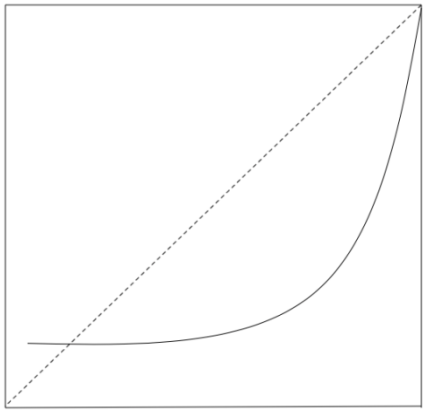
$$\begin{cases} V(X+Y) < V(X) + V(Y) \\ V(X+Y) > V(X) + V(Y) \end{cases}$$

Value function might be understood as convex function when investor confront loss and as concave function when investors gain profits. In the other world, investors showing a strong dislike of risk and avoiding risk in profit period. By contrast, if investors had already suffered loss, they are more likely like risk. In summary, changes in investor psychology will adjusting with the profitability. The function has following features:



a) Value function divided into two areas are Profit area and loss area. Price function is a curve that crossing the origin and shows monotonically decreasing trend. Generally, return changes has positive impact on value.
b) The profit area is concave to avoid risks. However, loss area is under convex to seek risks.
c) Compared with slope of profit area and loss area. Loss area slope is palpable larger than profit area. This means if investors loss, they will generate negative utility.

As for decision weight function, it not belong to objective probability but it has closely connected with objective probability. It not only the previous event similarity function, but also measures the effect of time on individual aspirations. Decision weight function is a nonlinear function that probability is p. Function features as follows:



d) It might overestimate small probability event and expand benefits due to contingency. Therefore, it increases the averse degree of extreme losses.
e) It has subadditivity means small probability plays a great influence and p is limited in a specific range. In addition, if increase probability, the weight of probability will exceed multiple of decision weight probability. It might result overpricing in actually.

2.2.2 Safety first theory

Roy (1952) pointed the theory that investment portfolio might not exist risk-free risk bond in order to minimize the probability of bankruptcy for investors. It means investment portfolio need to construct with risky assets. Roy (1952) also hold the view that safety first theory based on normal distribution model and obey normal distribution which return is μ_p, standard deviation isσ_p . Under this assumption, the objective function is:

$$Z = \frac{s - \mu_p}{\sigma_p}$$

It can be seen that critical point is equal to $\frac{s-\mu_p}{\sigma_p}$. In the other world, if investors wealth W is lower than this critical point, they might have bankruptcy risk. By contract, if investors wealth W is higher than this critical point, they not face bankruptcy risk.

2.2.3 Safety, potential and expectation theory

Lopes (1986) based on safety first theory and enhanced point safety, potential and expectation theory (SP/A).

Lopes (1986) believe risk attitude “afraid” and “hope” are two factors impact on wealth expected E(W). Under risk attitude “afraid”, investors tend to endue high proportion and low wealth level to calculate E(W). While risk attitude “hope” will appear small weight and large wealth level to measure E(W). Assuming wealth is (i=1,2,3,4……), W is ranging from low to high $w_1 < w_2 < \ldots < w_n$and corresponding probability is p_i. The formula is:

$$E(w) = \sum p_i w_i$$

$$E(w) = \sum D_i(w_i - w_{i-1})$$

Using $h_s(D)$ and $h_p(D)$ to represent psychological “afraid” and “hope”, the function expression is:

$$h_s(D) = D^{(1+q_s)}(q > 0)$$

$$h_p(D) = 1 - (1 - D_p)^{1+q_p}(q_p > 0)$$

Comprehensive psychological “afraid” and “hope”, the function h(D) expression is:

$$h(D) = \delta h_s(D) + (1 - \delta)h_p(D)$$

According to this formula, risk attitude “afraid” investors $E_h(W)$ is lower than E(W) , $E_h(W)$will decrease with the increase of degree of “afraid” psychological.

In SP/A theory, investors utilize $E_h(W)$ to transfer E(w) and use $r_i = h(D_{i+1}) - h(D_i)$.

Investors purpose can be expressed as：

$$max\mu(E_h(w), D(s))$$

In summary, SP/A theory has a significant role in risk evaluating and taking psychological factors into consideration, using quantifying method to analysis risk measurement. However, the disadvantage is the predicted data are difficult to observe and may not utilize in real life.

3.Security market risk measurement theory research

3.1New measure of securities risk and structure mathematical model

3.1.1 Establish mathematical model with no income reinvested

Under behavior finance, assuming people has two psychological account. One is psychological up line suppose expectation return will rising and another is psychological down line suppose expectation return will declining in the future. In order to facilitate the description, introduce some symbols: assuming has securities n that represent sample. P is security price and corresponding price can be expressed as$P_1, P_2 \ldots P_n$. Supposing psychological lines has two direction up and down. In addition, price has positive relationship with psychological. Psychological lines slope as follow:

$$k = \frac{\Delta y}{\Delta x} = \frac{1}{n-1}\sum P_t$$

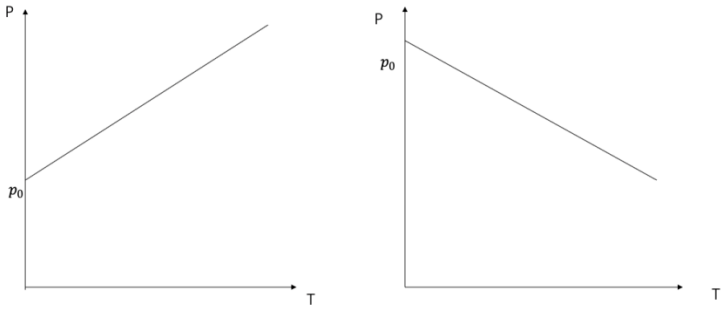
psychological line function is:

$$P = \alpha + kt = P_0 + kt$$

Based on risk feature uncertainty and lossy. Mixing with investors psychological such as avoid risk. Psychological line has positive slope and intercept is $\frac{\Delta p}{\Delta T}$. If securities located in this curve means this security is zero risk. Risk measured as follows:

$$Risk = \sqrt{\sum (P - (P_0 + kT))^2} = 0$$

When $k \geq 0$, $k < 0$, psychological up line and psychological down line pictures are:



The slope also can be expressed as:

$$k = \frac{\Delta y}{\Delta x} = \frac{1}{n-1}\sum P_t$$

Because investors psychological influence, the loss is unlimited and if securities located in this line indicates this security has endless risk. Risk can be measured as follow:

$$Risk = \frac{1}{\sqrt{\sum * (P - (P_0 + kT))^2}} = \infty$$

3.2.1 Establish mathematical model with income reinvested

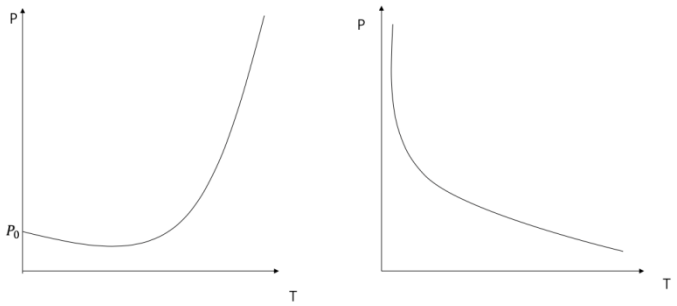
Without income reinvestment model is like a linear equation of one variable and slope is k. However, if consider with income reinvested, introduce some symbols: assuming has securities n that represent sample. P is security price and corresponding price can be expressed as $P_1, P_2, \ldots P_n$.The return formula is: $\frac{P_n - P_{n-1}}{P_{n-1}}$, In conclusion, psychological line is more likely a multivariate equation and slope as follow:

$$k = \sqrt[n-1]{\Pi(\frac{P_n}{P_{n-1}})} - 1$$

$$P = P_0 * (k+1)^n$$

$$Risk = \sqrt{\sum (P_t - P)^2} = 0$$

When $k \geq 0$, < 0 , psychological up line and psychological down line pictures are:



$$Risk = \frac{1}{\sqrt{\sum (P_t - P)^2}} = \infty$$

4. Research conclusions

In summary, this paper compared the difference evaluation methods under traditional finance and behavior finance to verify risk and securities return under new risk measurement method accuracy is higher than variance method.

In addition, new risk measurement approach overcomes variance method disadvantages and considers investors psychology account and other subjective factors.

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